MEDICAL IMAGE COMPRESSION

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Medical Computing course
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papers:
- Introduction to wavelet-based compression of medical images
- A Comparison of JPEG and Wavelet Compression Applied to Computed Tomography Brain, Chest, and Abdomen Images

Why Compression?

- Storage (archive)
  - Tera bytes of images per institute per year
- Bandwidth (telemedicine)
  - Transmitting many times the original data

Multimedia Data	Size/Duration	Bits/Pixel or Bits/Sample	Uncompressed Size (B for bytes)	Transmission Bandwidth (b for bits)	Transmission Time (using a 28.8K Modem)
A page of text	11" x 8.5"	Varying resolution	4-8 KB	32-64 Kb/page	1.1 - 2.2 sec
Telephone quality speech	10 sec	8 bps	80 KB	64 Kb/sec	22.2 sec
Grayscale Image	512 x 512	8 bpp	262 KB	2.1 Mb/image	1 min 13 sec
Color Image	512 x 512	24 bpp	786 KB	6.29 Mb/image	3 min 39 sec
Medical Image	2048 x 1680	12 bpp	5.16 MB	41.3 Mb/image	23 min 54 sec

Compression - how?

- Lossless
  - Preserve image quality
  - Low compression ratio (3:1 4:1)
- Lossy
  - Loss of details
  - High compression ratio (10:1 100:1)

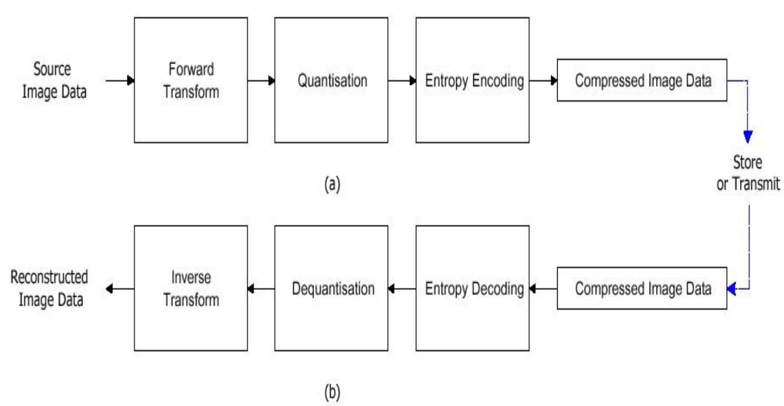
Lossless Compression

- RLE – Run Length Encoding
  - Repeated elements
- Entropy – Huffman, Arithmetic
  - Variable length code
- LZW – Lempel, Ziv, Welch
  - Repeated patterns
- And more

Lossy Compression

- Motivation – Compression ratio
- What kind of information can we afford to lose?
  Visually redundant (high frequencies?)
- Example:

General scheme for lossy compression



Lossy Compression

- The most prominent two schemes are:

- JPEG – Joint Photographic Expert Group
  - Extension: .jpg/.jpeg

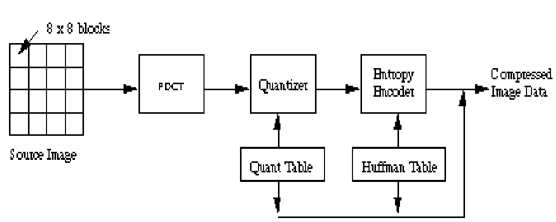
- Wavelets (small waves) – JPEG2000
  - Extension: .jp2

- Both part of DICOM standard

  (**D**igital **I**maging and **C**ommunications in **M**edicine)

JPEG

- DCT based (Discrete Cosine Transform)
  - A close relative of DFT (no complex numbers)
  - Transforms the data into the frequency domain
- Blocks of 8x8
  - Introduces blocking artifacts
- Scheme:



JPEG Example:

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

Original - 128 →

$$\begin{bmatrix} -76 & -73 & -67 & -62 & -58 & -67 & -64 & -55 \\ -65 & -69 & -73 & -38 & -19 & -43 & -59 & -56 \\ -66 & -69 & -60 & -15 & 16 & -24 & -62 & -55 \\ -65 & -70 & -57 & -6 & 26 & -22 & -58 & -59 \\ -61 & -67 & -60 & -24 & -2 & -40 & -60 & -58 \\ -49 & -63 & -68 & -58 & -51 & -60 & -70 & -53 \\ -43 & -57 & -64 & -69 & -73 & -67 & -63 & -45 \\ -41 & -49 & -59 & -60 & -63 & -52 & -50 & -34 \end{bmatrix}$$

($x \longrightarrow$ across, $y \downarrow$ down)

DCT ↓

$$X_{k_1,k_2} = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x_{n_1,n_2} \cos\left[\frac{\pi}{N_1}\left(n_1+\frac{1}{2}\right)k_1\right] \cos\left[\frac{\pi}{N_2}\left(n_2+\frac{1}{2}\right)k_2\right].$$

$$\begin{bmatrix} -415 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix}$$

($u \longrightarrow$ across, $v \downarrow$ down)

← Quantization

$$\begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -4 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Quantization table:

$$\begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

JPEG Example:

$$\begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -4 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

De-quantization →

$$\begin{bmatrix} -416 & -33 & -60 & 32 & 48 & -40 & 0 & 0 \\ 0 & -24 & -56 & 19 & 26 & 0 & 0 & 0 \\ -42 & 13 & 80 & -24 & -40 & 0 & 0 & 0 \\ -56 & 17 & 44 & -29 & 0 & 0 & 0 & 0 \\ 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Inverse DCT ↓

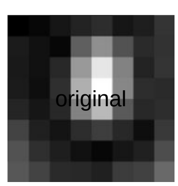
$$\begin{bmatrix} -68 & -65 & -73 & -70 & -58 & -67 & -70 & -48 \\ -70 & -72 & -72 & -45 & -20 & -40 & -65 & -57 \\ -68 & -76 & -66 & -15 & 22 & -12 & -58 & -61 \\ -62 & -72 & -60 & -6 & 28 & -12 & -59 & -56 \\ -59 & -66 & -63 & -28 & -8 & -42 & -69 & -52 \\ -60 & -60 & -67 & -60 & -50 & -68 & -75 & -50 \\ -54 & -46 & -61 & -74 & -65 & -64 & -63 & -45 \\ -45 & -32 & -51 & -72 & -58 & -45 & -45 & -39 \end{bmatrix}$$

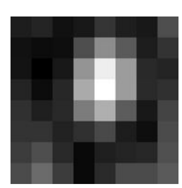
← +128

$$\begin{bmatrix} 60 & 63 & 55 & 58 & 70 & 61 & 58 & 80 \\ 58 & 56 & 56 & 83 & 108 & 88 & 63 & 71 \\ 60 & 52 & 62 & 113 & 150 & 116 & 70 & 67 \\ 66 & 56 & 68 & 122 & 156 & 116 & 69 & 72 \\ 69 & 62 & 65 & 100 & 120 & 86 & 59 & 76 \\ 68 & 68 & 61 & 68 & 78 & 60 & 53 & 78 \\ 74 & 82 & 67 & 54 & 63 & 64 & 65 & 83 \\ 83 & 96 & 77 & 56 & 70 & 83 & 83 & 89 \end{bmatrix}$$

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

Original





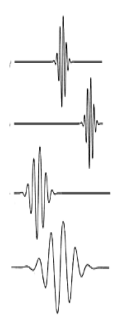
Results

- From high to low quality (low to high compression ratio):

The wavelet transform

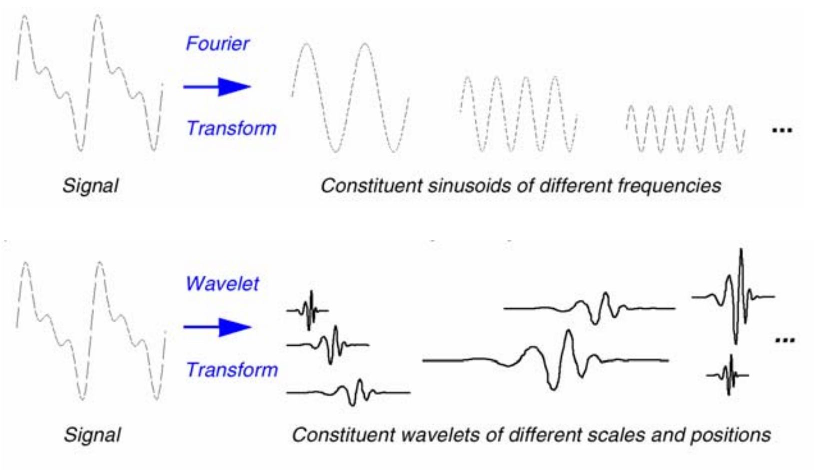
A “**<u>mother wavelet</u>**”
is a function located in time and frequency.

The function is dilated (scaled) and
time-shifted (translated) to form a set of functions:



DWT – Discrete Wavelet Transform
The signal is expressed as a collection
of waveletes in different positions and scales

Fourier vs. Wavelets decomposition



Wavelets as filters

- Essentially a multiresolution filtering process:
  - HPF (Mother wavelet)
  - LPF (Father scaling)
  - Apply in different resolutions

One Stage Filtering

Approximations and details:

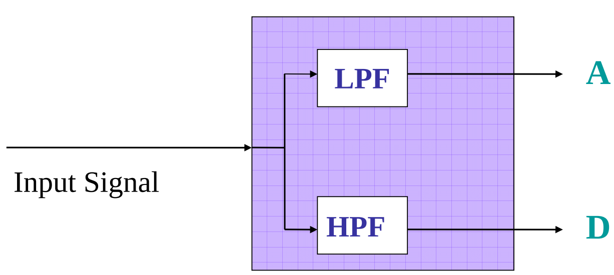
- The low-frequency content is the most important part in many applications, and gives the signal its identity.

  This part is called "*Approximations*"

- The high-frequency gives the 'flavor', and is called "*Details*"

Approximations and Details:

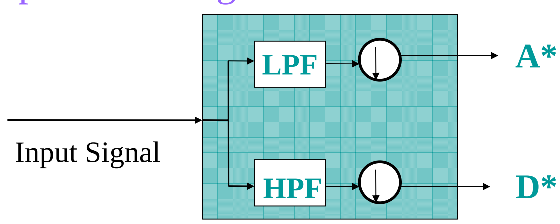
- **Approximations**:low- frequency components of the signal
- **Details**: high frequency components



Decimation

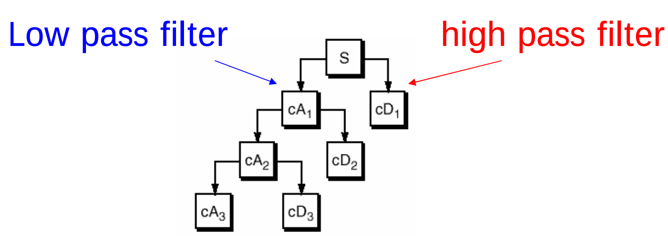
- The former process produces <u>twice the data</u>
- To correct this, we *Down sample* (or: *Decimate)* the filter output by two.

A complete one stage block :



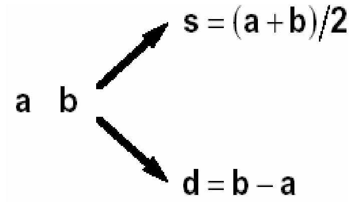
Multi-level Decomposition

- Iterating the decomposition process, breaks the input signal into many lower-resolution components: *Wavelet decomposition tree*:



A Simple Example: The Haar Wavelet

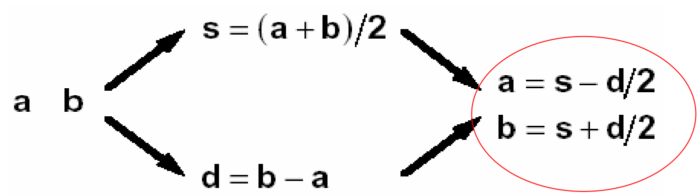
- Consider two neighboring samples a and b of a sequence.
  - a and b have some correlation.
  A simple linear transform:



- High correlation
  - small |d|, fewer bits representation.
  (i.e. a=b,d=0)

The Haar Wavelet Con't

- No loss of any information
- Reconstruction formula of a and b:



The key behind *Haar* Wavelet Transform:
these reconstruction formulas can be found by inverting a 2x2 matrix.

The Haar Wavelet Con't

- Signal S_n of 2^n sample values $S_{n,i}$:

$$S_n = \{S_{n,i} \mid 0=< i <= 2^n\}$$

- Apply average (S_{n-1}) and difference (d_{n-1}) transform onto each pair:

a = S_{2i}, b = S_{2i+1}
- 2^{n-1} pairs (i=0…2^{n-1})

$$S_{n-1,i} = \frac{S_{n,2i}+S_{n,2i}}{2}$$

$$d_{n-1,i} = S_{n,2i+1} - S_{n,2i}$$

- Recover the original signal S_n from S_{n-1} and d_{n-1}

The Haar Wavelet Con't

- S_{n-1} as Approximations
- d_{n-1} as Details

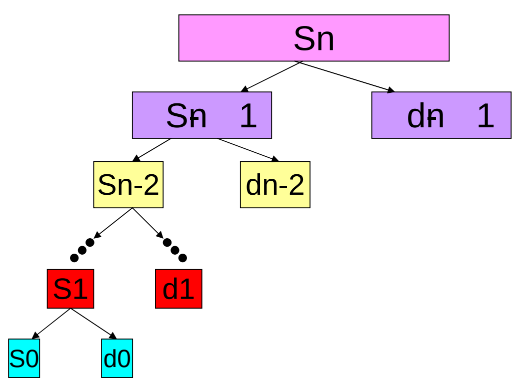
- Signal with local coherence
  - approximations closely resembles the original signal
  - detail is very small (efficient representation)

The Haar Wavelet Con't

- Applying the same transform (averages and differences) to S_{n-1} itself.
- Split S_{n-1} to (yet) coarser signal S_{n-2} and another difference signal d_{n-2},each of them contain 2^{n-2} samples.
- We can repeat this transform n times till S_0 contains only one sample $S_{0,0}$.

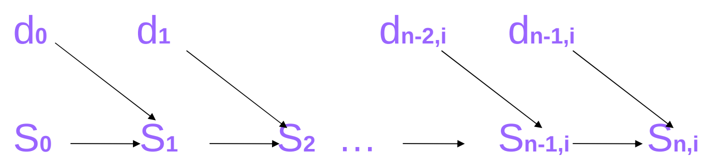
This is the Haar transform

The Haar **transform**



Structure of the wavelet transform: recursively split into averages and differences

The Haar **transform**



- Structure of the inverse wavelet transform: recursively merge averages and differences.

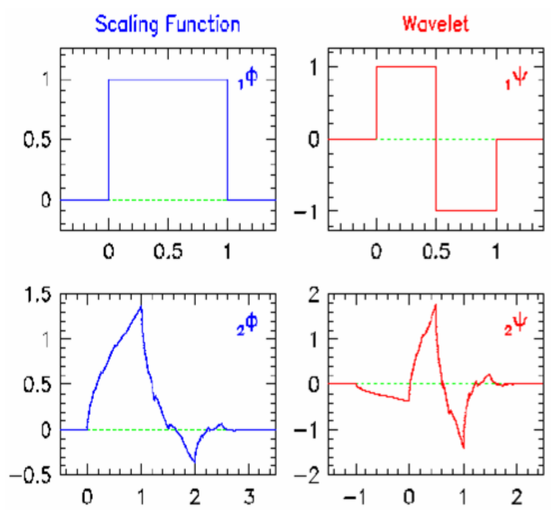
The Haar **transform Con't**

- The cost of computing the transform is O(N).

(FFT cost is O(NlogN))

Wavelet functions examples

- Haar function
- Daubechies function



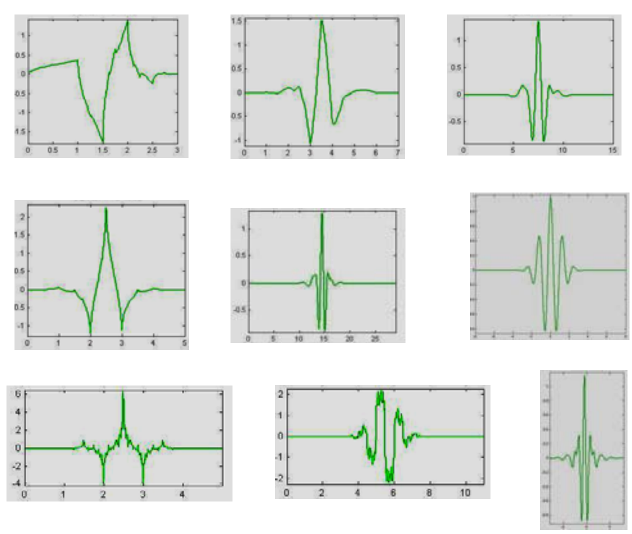
Symlet

Coiflet

Biorthogonal Splines

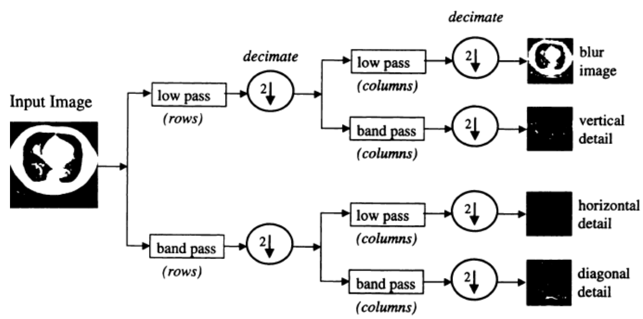
Morlet

Meyer



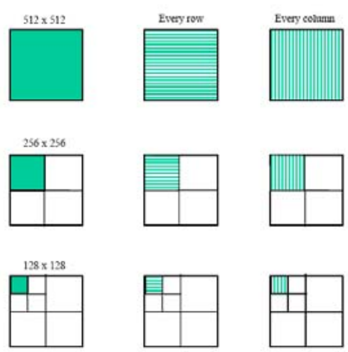
Wavelets Image decomposition

- One level decomposition

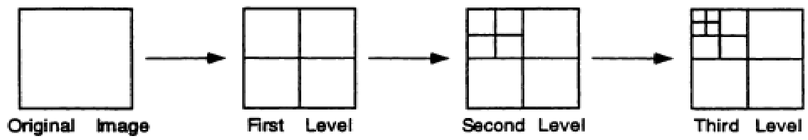


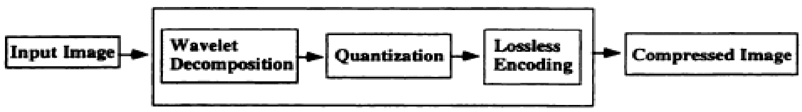
Wavelets Image decomposition

- Multi level decomposition



Decomposition and Compression





Quantization

<u>two important observations:</u>

1. Natural images in general have a low pass spectrum, so the wavelet coefficients will, on average, be smaller in the higher subbands than in the lower subbands.

631	544	86	10	-7	29	55	-54
730	655	-13	30	-12	44	41	32
19	23	37	17	-4	–13	-13	39
25	-49	32	-4	9	-23	-17	-35
32	-10	56	-22	-7	-25	40	-10
6	34	-44	4	13	-12	21	24
-12	-2	-8	-24	-42	9	-21	45
13	-3	-16	-15	31	-11	-10	-17

typical wavelet coefficients for a 8*8 block in a real image

2. Large wavelet coefficients are more important than smaller wavelet coefficients.

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Quantization

- At low bit rates a large number of the transform coefficients are quantized to zero (Insignificant Coefficients).

Quantized Coefficients

64	56	48	32	24	16	0	0
56	48	40	24	16	23	0	0
40	40	30	24	16	8	0	8
32	32	32	24	24	16	0	0
24	24	16	8	0	0	8	0
16	16	8	0	0	8	0	0
0	0	0	8	0	0	0	0
0	0	0	0	0	0	0	0

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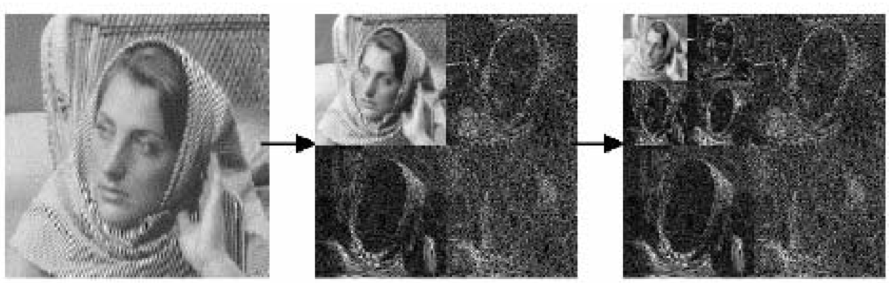
Example

- Here is a two-stage wavelet decomposition of an image. Notice the large number of zeros (black):



Matlab toolbox

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Comparison

- Between DCT & DWT on sequences of medical images

- Brain, Chest and Abdomen:

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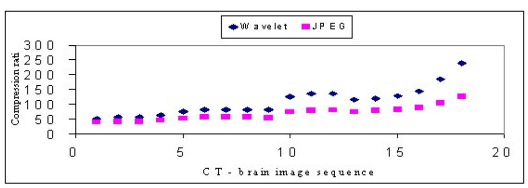
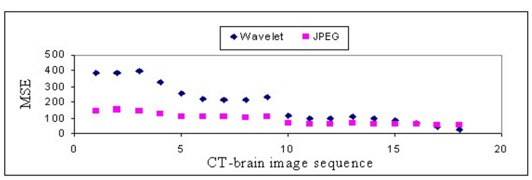
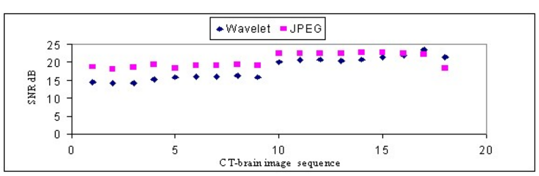
Comparison – Brain







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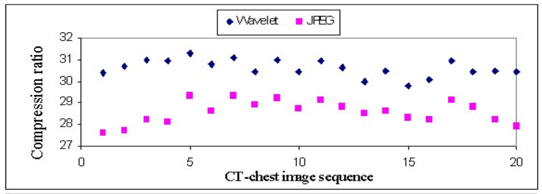
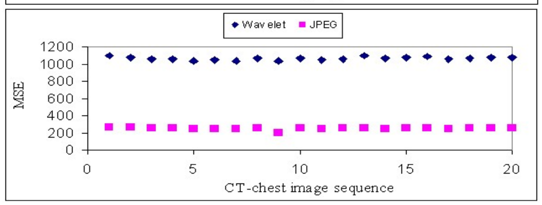
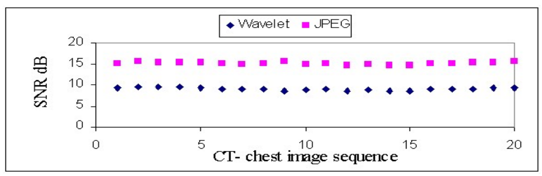
Comparison – Chest







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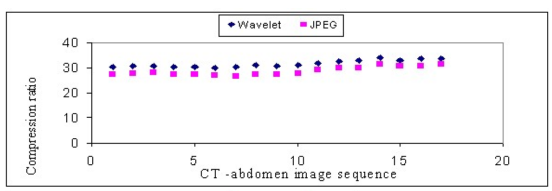
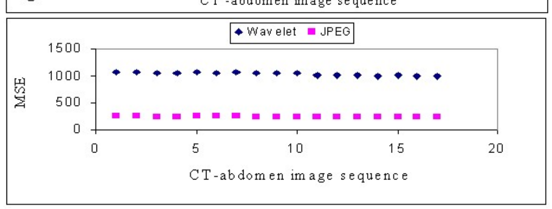
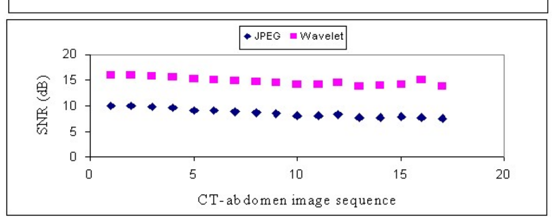
Comparison – Abdomen







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Comparison

JPEG JPEG2000

Reconstructed images compressed at 0.25 bpp

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Comparison

(a)
JPEG

(b)
JPEG2000

Reconstructed images compressed at 0.125 bpp

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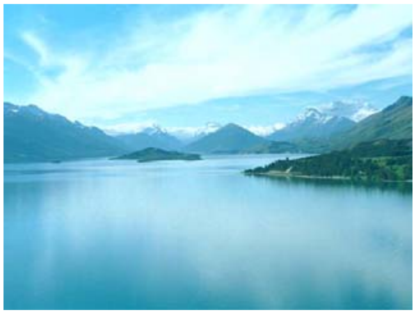
Comparison

JPEG 2000 (1.83 KB)



Original (979 KB)

JPEG (6.21 KB)

Conclusions

- DWT outperforms DCT at high compression ratios and is (a bit) faster
- Depends on the image – anatomic structures, complexity of diagnostic information
- Careful consideration must be given to the level of compression ratio before archiving clinical images otherwise essential information will be lost
- Wavelets in other areas (e.g. epilepsy)

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